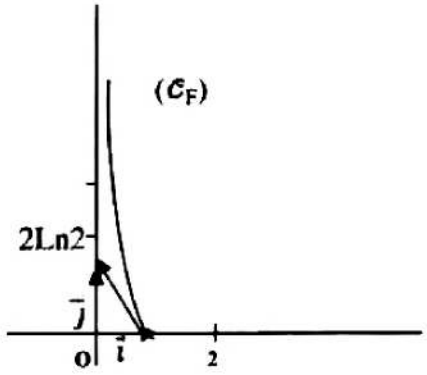
$F(1) = \int_1^1 f(t)\,dt = 0$



$F_g'(1) = -2\,Ln2$

C) $x \in]2, +\infty[$; $n \in IN$

$$g_n(x) = \int_{2\sqrt{2}}^{x} \frac{t^{2n}}{\sqrt{t^2-4}}\,dt$$

$$\ell_n = \lim_{x \to 2^+} g_n(x)$$

1) la fonction : $t \mapsto \frac{t^{2n}}{\sqrt{t^2-4}}$ est continue

Sur $]2, +\infty[$ et $2\sqrt{2} \in]2, +\infty[$ d'où l'existence de $g_n(x)$ pour $x \in]2, +\infty[$

2) $$g_0(x) = \int_{2\sqrt{2}}^{x} \frac{1}{\sqrt{t^2-4}}\,dt$$

$$= \int_{2\sqrt{2}}^{x} f'(t)\,dt$$

$$= [f(t)]_{2\sqrt{2}}^{x} = f(x) - f(2\sqrt{2})$$

$$g_0(x) = Ln(x + \sqrt{x^2-4}) - Ln(2+2\sqrt{2})$$

$$\ell_0 = \lim_{x \to 2^+} g_0(x) = Ln2 - Ln(2+2\sqrt{2})$$

$$\ell_0 = -Ln(1+\sqrt{2})$$

3) $$g_1(x) = \int_{2\sqrt{2}}^{x} \frac{t^2}{\sqrt{t^2-4}}\,dt$$

On pose :

$$\begin{cases} u'(t) = \frac{t}{\sqrt{t^2-4}} \longrightarrow u(t) = \sqrt{t^2-4} \\ v(t) = t \longrightarrow v'(t) = 1 \end{cases}$$

$$g_1(x) = \left[t\sqrt{t^2-4}\right]_{2\sqrt{2}}^{x} - \int_{2\sqrt{2}}^{x} \sqrt{t^2-4}\,dt$$

$$\int_{2\sqrt{2}}^{x} \sqrt{t^2-4}\,dt = \int_{2\sqrt{2}}^{x} \frac{t^2-4}{\sqrt{t^2-4}}\,dt$$

$$= \int_{2\sqrt{2}}^{x} \frac{t^2}{\sqrt{t^2-4}}\,dt - 4\int_{2\sqrt{2}}^{x} \frac{dt}{\sqrt{t^2-4}}$$

$$\int_{2\sqrt{2}}^{x} \sqrt{t^2-4}\,dt = g_1(x) - 4\,g_0(x)$$

D'où

$$g_1(x) = \left[t\sqrt{t^2-4}\right]_{2\sqrt{2}}^{x} - g_1(x) + 4\,g_0(x)$$

$$\Rightarrow 2\,g_1(x) = x\sqrt{x^2-4} - 4\sqrt{2} + 4\,g_0(x)$$

$$\Rightarrow g_1(x) = \frac{x}{2}\sqrt{x^2-4} - 2\sqrt{2} + 2\,g_0(x)$$

$$\ell_1 = \lim_{x \to 2^+} g_1(x)$$

$$= \lim_{x \to 2^+} \left(\frac{x}{2}\sqrt{x^2-4} - 2\sqrt{2} + 2\,g_0(x)\right)$$

$$= -2\sqrt{2} + 2\,\ell_0$$

$$\ell_1 = -2\sqrt{2} - 2.Ln(1+\sqrt{2})$$

4) – a)

$$g_{n+1}(x) = \int_{2\sqrt{2}}^{x} \frac{t^{2n+2}}{\sqrt{t^2-4}}\,dt$$

on pose

$$\begin{cases} u(t) = \frac{t}{\sqrt{t^2-4}} \longrightarrow U(t) = \sqrt{t^2-4} \\ v(t) = t^{2n+1} \longrightarrow v'(t) = (2n+1)t^{2n} \end{cases}$$

Qarray.tn - Aqra Blech - Qarray.tn